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# Identity

## Information Identity

In information philosophy, identity depends on the *total information* in an object or concept.

We distinguish the *intrinsic* information inside the object (or concept) from any relational information with respect to other objects that we call *extrinsic* or external information. We can “pick out” the intrinsic information as that which is “self-identical” in an object. The Greeks called this the  $\pi\rho\delta\varsigma\ \acute{\epsilon}\alpha\nu\tau\omicron$  - self-relation, or  $\iota\delta\iota\omicron\varsigma\ \pi\omicron\iota\omicron\nu\omicron$ , “peculiar qualifications” of the individual.

Self-identity, then, is the simple fact that the *intrinsic* information and the *extrinsic* relational or dispositional information are unique to this single object. No other object can have the same disposition relative to other objects. This is an absolute kind of identity. Some metaphysicians say that such identity is logically necessary. Some say self-identity is the only identity, but we can now support philosophers who argue for a *relative identity*.

To visualize our concept of information identity, imagine putting yourself in the position of an object. Look out at the world from its vantage point. No other object has that same view, that same relation with the objects around you, especially its relation with you. Now another object could have intrinsic information identity. We will identify a very large number of objects and concepts in the world that are intrinsically identical, including natural and artifactual kinds, which we may call digital kinds, since they are identical, bit for bit. This is *relative identity*.

## A Criterion for Identity

After accepting the fundamental fact that nothing is perfectly identical to anything but itself, the criterion for *relative identity*, for identical “in some respect,” or *qua* that respect, is that some subset of the information in two different things must be the same information, bit for bit.



*Relative identity* means that  $a$  can be the same  $I$  as  $b$ , but not the same  $E$  as  $b$ , where  $I$  is the sum of all the *intrinsic* properties and relations - internal self-relations between an object's different parts. For physical objects, these could be within some physical boundary, the borderlines subject to conditions of *vagueness*. In a biological entity, intrinsic information includes the vast communications going on inside and between the cells, which makes it much more than a mereological sum of its material parts.

The  $E$  for an object is the sum of *extrinsic* relations an object has with things outside, including its disposition in space and time.

Mathematically,  $\int_i F(x) = \int_i G(x)$ , but  $\int_e F(x) \neq \int_e G(x)$ , which says that  $F(x)$  and  $G(x)$  are identical over their intrinsic domains (i) but differ over their extrinsic domains (e).

Set theoretically, in classical propositional calculus, we can say that  $I_a$  is the set of intrinsic properties and relations that can be predicated in propositions about an object  $a$ .  $E_a$  is the set of extrinsic relations. We can now describe why *absolute* identity is limited to *self-identity*.

If  $I_a + E_a = I_b + E_b$ , then  $a$  and  $b$  are one and the same object.

And, if  $Ia = Ib$ , then  $a$  and  $b$  are relatively identical, *qua* their information content.

Note that while self-identity is reflexive, symmetric, and an equivalence self-relation, relative identity is often none of these. This is because, unlike MAX BLACK's identical spheres, SAUL KRIPKE's natural kinds, and our many digital clones, some part of the information in  $a$  and  $b$  may be identical, but the information that is not identical may also differ in quantity. We can say that if  $aRb$  is 60% identical,  $bRa$  may be only 10% identical.

The application of this criterion is the quantitative analysis, the quantification, of the total information in and about both objects.

Extensional *quantification* over things in analytic language philosophy is about their set membership, which is dependent on language references to the properties of objects.



By contrast, quantification in information philosophy is a calculation of the total information content in the entities, in principle free of language ambiguities, but in practice, very difficult.

### A Criterion for Essence

Information identity suggests a possible definition of the “*essence*” of an object, what is “*essential*” about it. Furthermore, if two objects are considered “*essentially*” the same, we can pick out the subset of information that corresponds to that “*essence*.”

A subset of the intrinsic information may be essential with respect to (*qua*) some concept of the object. As EDMUND HUSSERL emphasized, our concepts about objects depend on our intentions, our intended uses of the object, which give it different (pragmatic) meanings. We can say that an essence is the subset of an object’s information that is isomorphic to the information in the concept.

What we call a “concept” about a material object is usually some subset of the information in the object, accurate to the extent that the concept is isomorphic to that subset. By “picking out” different subsets, we can sort objects. We can compare objects, finding them similar *qua* one concept and different *qua* another concept. We can say, for example, that “ $a = b$ ” *qua* color but not *qua* size.

But there are concepts that may have little to do with the intrinsic peculiar information about an object. They are concepts imposed on the object by our intended uses of it.

We must distinguish these extrinsic essences – our external ideas and concepts about what the object is – from the intrinsic essences that depend only on the object itself and its own purposes, if any. The essences we see in an object are subjective, but we may define an objective essence as the total intrinsic information, including internal messaging, in the object.

Husserl and GOTTLÖB FREGE both pointed out that our Ideas are dependent on our personal experience. Experience constrains and amplifies our possible concepts. Two persons may get the general “sense” or “meaning” of something referred to, but Frege said the “idea” or representation (*Vorstellung*) in each mind can



be very different, based on that individual's experience. Information philosophy locates the creation of meaning in the responses of the *experience recorder and reproducer* (ERR) to different stimuli.

The relation "identical to," between two numerically distinct concrete or abstract entities, is the source of logical puzzles and language games through the ages that are little more than verbal disputes. Most such disputes are easily resolved or "*dis-solved*" by paying careful attention to all the information, all the particular properties, *intrinsic* and *extrinsic*, of the two entities that may be identical *qua* some particular properties.

### Background of the Problem

Identity has been a major problem in philosophy and metaphysics since the Ancients. Even Plato wondered whether two things could be identical:

**"Socrates.** It is in your opinion, possible for the mind to regard one thing as another and not as what it is **Theaetetus.** Yes, it is.

**Socrates.** Now when one's mind does this, does it not necessarily have a thought either of both things together or of one or the other of them? **Theaetetus.** Yes, it must; either of both at the same time or in succession.

**Socrates.** Then whenever a man has an opinion that one thing is another, he says to himself, we believe, that the one thing is the other. **Theaetetus.** Certainly."<sup>1</sup>

And here is Aristotle:

"The same" means (a) accidentally the same...For it is not true to say that every man is the same as "the cultured"; because universal predications are essential to things, but accidental predications are not so, but are made of individuals and with a single application. ...

Some things are said to be "the same" in this sense, but others (b) in an essential sense, in the same number of senses as "the one" is essentially one; for things whose matter is formally or numerically one, and things whose substance is one, are said to be the same. Thus "sameness" is clearly a kind of unity in the being, either of two or more things, or of one thing treated as more

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1 Plato, *Theaetetus*, 189D-190B



than one; as, e.g., when a thing is consistent with itself; for it is then treated as two.

Things are called “other” of which either the forms or the matter or the definition of essence is more than one; and in general “other” is used in the opposite senses to “same.”

Things are called “different” which, while being in a sense the same, are “other” not only numerically, but formally or generically or analogically; also things whose genus is not the same; and contraries; and all things which contain “otherness” in their essence.”<sup>2</sup>

The fundamental notion of *identity* refers only to the substance and the bundle of *intrinsic* properties (the material substrate and the *immaterial* form) of a single entity. Literally and etymologically it is “id-ENTITY,” same entity, from Latin *idem*, “same,” and *entitas*.

In Greek, self-identity is the *idios*, one’s personal, private, peculiar (intrinsic) properties, separate and distinct from the (extrinsic) properties of others and one’s relational properties to others. From Aristotle to the Stoics, Greek philosophers distinguished the individual’s *material* substance from the *immaterial* “peculiar qualifications” of the individual. They were accused by the Academic Skeptics of seeing two things - coinciding objects<sup>3</sup> - where there is only one, but they were only distinguishing the form of an object from its matter.

The Stoic term for “constituent substance” or substrate, following Aristotle, was *ὑποκείμενον* (“the underlying”). Their term for the unique person, possibly separate from the material body, was *πρὸς ἑαυτο* - self-relation, or *ἰδιος ποιὸν* - the peculiar qualifications of a particular individual “who,” for example, Socrates, as opposed to *κοινός ποιὸν*, a general “whoness,” for example, a human being.

The Greeks also carefully distinguished relational or dispositional properties that depend on an individual’s position in space and time or its causal interactions with other individuals. They called these the *pōs ti alla* or *pōs echon*, usually translated as the relatively dispositional qualifications.<sup>4</sup>

2 Aristotle, *Metaphysics*, V, ix, 1018b

3 See chapter 7.

4 Long and Sedley (1989) *The Hellenistic Philosophers*, vol 1. p.163



Ignoring this important ancient distinction between intrinsic and merely extrinsic properties (for example, a name) is the source of many confusions in modern identity theory.

Since the seventeenth century, logicians following GOTTFRIED LEIBNIZ have held that necessary truths, including *a priori* and *analytic* truths, have the unique property of being “true in all possible worlds.”

Recently, identity figured prominently in discussions of possible worlds. In the 1940’s, the concepts of necessity and possibility were added to symbolic logic. Surprisingly, the modal logicians claimed that if two things are identical, they are *necessarily* identical. Does the modal logic proof of the necessity of identity allow us to know something about possible worlds? This is the claim of SAUL KRIPKE and DAVID LEWIS.

It is a sad fact that the addition of modality found little evidence for the importance of possibilities, let alone contingency, which describes almost everything that is the case in our actual world. The possible worlds of Lewis (and perhaps Kripke?) appear to be *eliminatively materialist* and *determinist*, with no real *contingency*.

Is there a sense in which two numerically distinct objects can be identical? Can one of these be in another possible world, what Lewis calls a *counterpart* object? Metaphysicians puzzle over this and a related question, can two things be in the very same place at the same time as coinciding objects? Many metaphysical puzzles and paradoxes start with this flawed assumption.

With information as our analytic tool, we can show that two things that share every property, *intrinsic* internal properties and *extrinsic* external relations with all the other objects in the world, including their positions in space and time, can only be perfectly identical if they are *actually one and the same object*. It seems fine to say that any thing is necessarily itself. We can also show that two things sharing intrinsic internal properties are *relatively* identical.

Leibniz and Gottlob Frege both said clearly that two objects claiming to be identical are one object under two names. A large fraction of the metaphysical literature still ponders this question, (e.g., Hesperus and Phosphorus as two names for Venus).



Absolute identity is simply the relation that any thing has with itself. *Everything is identical to itself*. Anything else is merely “*relative identity*,” identical in some respect (*qua*).

Self-identity is a monadic property that applies only to the object itself. Many modal logicians (starting with RUTH BARCAN MARCUS, DAVID WIGGINS, and SAUL KRIPKE) mistakenly thought that given two “identical” objects  $x$  and  $y$ ,  $x$ 's property of being equal to  $x$  ( $x = x$ ) can be a property of  $y$  ( $= x$ ). Information philosophy shows this is only the case if  $x$  and  $y$  are the same object. Numerically distinct  $x$  and  $y$  can only have a *relative* identity.

LUDWIG WITTGENSTEIN described this in *Tractatus* 5.5303,  
 “Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing.”

## Leibniz

Most of the metaphysical problems of identity, and especially recent claims about the necessity of identity, can be traced back to the great rationalist philosopher GOTTFRIED LEIBNIZ, who argued for the replacement of ordinary language with a *lingua characterica*, an ambiguity-free set of terms that would eliminate philosophical puzzles and paradoxes. BERTRAND RUSSELL, Ludwig Wittgenstein, and RUDOLF CARNAP all believed in this Leibnizian dream of ambiguity-free, logically true, facts about the world that may be true in all possible worlds.

Unfortunately, fundamental limits on logic and language such as the Gödel and Russell paradoxes have prevented Leibniz's ideal ambiguity-free language, but many modern paradoxes, including questions about identity and necessity, are resolvable in terms of information, as we shall see.

Leibniz defined an “axiom of identity” as “everything is identical to itself.” He called it a “primary truth.” He said “There are no two individuals indiscernible from one another.” This is sometimes called Leibniz's Law, the *Identity of Indiscernibles*. “To suppose two things indiscernible is to suppose the same thing under two names,” thus introducing some puzzles about naming that have caused massive confusion in language philosophy and metaphysics for the past seven decades, notably in the work of WILLARD VAN ORMAN QUINE.





## Leibniz's Laws

More than any other philosopher, Leibniz enunciated clear principles about identity, including his *Identity of Indiscernibles*. If we can see no differences between things, they may be identical. This is an empirical fact, and must be tested empirically, as Leibniz knew.

But once again, whenever we are talking about two things, that there is a difference between them, a discernible difference, is transparently obvious. Two things are numerically distinct even if they have identical internal information.

Leibniz also described a corollary or converse, the *Indiscernibility of Identicals*.<sup>5</sup> But this idea is necessarily true, if such things as numerically distinct identical objects exist. We shall show that such things have only a *relative* identity, identity in some respects.

Leibniz anticipated the best modern efforts of analytical language philosophers like Frege's distinction between sense (meaning) and reference and Saul Kripke's odd idea that names are metaphysically necessary,<sup>6</sup> when we know well that words are arbitrary symbols.

Leibniz also gave us a *principle of substitutability* - "things are identical if they can be substituted for one another everywhere with no change in truth value."

Leibniz wrote:

"It is not true that two substances resemble each other entirely and differ in number alone."<sup>7</sup>

Indeed, every monad must be different from every other, For there are never in nature two beings which are precisely alike, and in which it is not possible to find some difference.<sup>8</sup>

There are no two individuals indiscernible from one another...

Two drops of water or milk looked at under the microscope will be found to be discernible.

To suppose two things indiscernible is to suppose the same thing under two names."<sup>9</sup>

5 so named by Quine (1943) "Notes on Existence and Necessity."

6 Kripke (1981) *Naming and Necessity*,

7 'Discourses on Metaphysics,' §9, in *Leibniz: Philosophical Writings*, p.19.

8 'Monadology,' §9, in *Leibniz: Philosophical Writings*, p.180

9 'Correspondence with Clarke,' in *Leibniz: Philosophical Writings*, p.216



## Frege

Gottlob Frege implemented Leibniz's program of a purely logical language in which statements or sentences with subjects and predicates are replaced with propositional functions, in which a term can be replaced by a variable. In modern terminology, the sentence Socrates is mortal can be replaced, setting the subject Socrates =  $x$ , and the predicate "is mortal" with  $F$ . " $x$  is  $F$ " is replaced by the propositional function  $Fx$ , which is read " $x$  is  $F$ ," or " $x$   $F$ 's."

Frege developed a calculus of these propositional functions, in which they are evaluated for their truth-functionality, using the formalism of Boole's two-valued logic. Frege also introduced quantification theory, replacing Aristotle's expression "for all" with a universal quantification operator, now written  $\forall x$  or  $(x)$ .

Frege repeated Leibniz's idea about identity and developed Leibniz's suggestion of one thing under two names, two distinct references. Where Leibniz had said, "To suppose two things indiscernible is to suppose the same thing under two names," Frege suggested that two names referring to the same thing can be in some respect "identical" because the thing they refer to is identical to itself.

"A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing. What is intended to be said by  $a = b$  seems to be that the signs or names " $a$ " and " $b$ " designate the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted... It would be mediated by the connection of each of the two signs with the same designated thing.

"If we found " $a = a$ " and " $a = b$ " to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its referent, i.e., its truth value. If now  $a = b$ , then indeed the referent of " $b$ " is the same as that of " $a$ ," and hence the truth value of " $a = b$ " is the same as that of " $a = a$ ." In spite of this, the sense of " $b$ " may differ from that of " $a$ ," and thereby the sense expressed in " $a = b$ " differs from that of " $a = a$ ." In that case the two sentences do not have the same cognitive value."<sup>10</sup>

10 Frege (1892) *Sense and Reference*, trans. P.Geach and M.Black (1952), p.230



## Names and Reference

Although Frege was very clear, generations of philosophers have obscured his clarity by puzzling over different names and/or descriptions *referring* to the same thing that may lead to logical contradictions – starting with Frege’s original example of the Morning Star and Evening Star as names that refer to the planet Venus. Do these names have differing cognitive value? Yes. Can they be defined *qua* references to uniquely pick out Venus. Yes. Is identity a relation? No. But the names are relations, words that are references to the objects. And words put us back into the *ambiguous* realm of language.

Over a hundred years of confusion in logic and language consisted of finding two expressions that can be claimed in some sense to be identical, but upon *substitution* in another statement, they do not preserve the truth value of the statement. Besides Frege, and a few examples from BERTRAND RUSSELL (“Scott” and “the author of Waverly,” “bachelor” and “unmarried man”), Willard Van Orman Quine was the most prolific generator of substitution paradoxes (“9” and “the number of planets,” “Giorgione” and “Barbarelli,” “Cicero” and “Tully,” and others).

Just as information philosophy shows how to pick out information in an object or concept that constitutes the “peculiar qualifications” that individuate it, so we can pick out the information in two designating references that provide what Quine called “purely designative references.” Where Quine picks out information that leads to contradictions and paradoxes (he calls this “referential opacity”), we can “qualify” the information, the “sense” or meaning needed to make them referentially transparent when treated “intensionally.”

Frege pointed out that the reference (a name) may not be the general “sense” that a person educated in the customary knowledge of their community may have in mind. Nor is this general sense the specific idea or representation that will actually come to mind. That will be different and dependent on the person’s experiences.



Peirce

Peirce wrote on identity some time in the late nineteenth century, already including Frege’s quantization and suggesting notation to express the identity of “second-intention” relations.

His papers did not appear until two decades after his death.

“§4. SECOND-INTENTIONAL LOGIC

398. Let us now consider the logic of terms taken in collective senses. Our notation, so far as we have developed it, does not show us even how to express that two indices,  $i$  and  $j$ , denote one and the same thing. We may adopt a special token of second intention, say  $1$ , to express identity, and may write  $1_{ij}^*$ . But this relation of identity has peculiar properties. The first is that if  $i$  and  $j$  are identical, whatever is true of  $i$  is true of  $j$ . This may be written

$$\prod_i \prod_j \{1_{ij}^* + x_i + x_j\}.$$

The use of the general index of a token,  $x$ , here, shows that the formula is iconical. The other property is that if everything which is true of  $i$  is true of  $y$ , then  $i$  and  $j$  are identical. This is most naturally written as follows: Let the token,  $q$ , signify the relation of a quality, character, fact, or predicate to its subject. Then the property we desire to express is

$$\prod_i \prod_j \sum_k (1_{ij} + q_{ki}^* q_{kj}^*).$$

And identity is defined thus

$$1_{ij} = \prod_k (q_{ki} q_{kj} + q_{ki}^* q_{kj}^*) \bullet$$

That is, to say that things are identical is to say that every predicate is true of both or false of both. It may seem circuitous to introduce the idea of a quality to express identity; but that impression will be modified by reflecting that  $q_{ki} q_{kj}$ , merely means that  $i$  and  $j$  are both within the class or collection  $k$ . If we please, we can dispense with the token  $q$ , by using the index of a token and by referring to this in the Quantifier just as subjacent indices are referred to. That is to say, we may write

$$1_{ij} = \prod x (x_i x_j + x_i^* x_j^*)." 11$$

Here we see  
Leibniz’s Law,  
just as it is  
presented in the  
*Principia  
Mathematica*

11 Peirce (1885) ‘Exact Logic’, *Collected Papers*, (1933) vol.III, p.233



Peirce also commented briefly on Leibniz's principle of the Identity of Indiscernibles.

“They are like two ideal rain drops, distinct but not different. Leibniz's “principle of indiscernibles” is all nonsense. No doubt, all things differ; but there is no logical necessity for it. To say there is, is a logical error going to the root of metaphysics; but it was an odd hodge-podge, Leibniz's metaphysics, containing a little to suit every taste.”<sup>12</sup>

## Principia Mathematica

It is in Bertrand Russell's *Principia Mathematica* that we first encounter identity theory written in symbolic logic terminology, using the mathematical sign of equality.

Part I, Mathematical Logic

Section B, Theory of Apparent Variables

### \*13. IDENTITY

The propositional function “x is identical with y” will be written “ $x = y$ .” We shall find that this use of the sign of equality covers all the common uses of equality that occur in mathematics. The definition is as follows:

\*13.01.  $x = y . = : (\varphi) : \varphi ! x . \supset . \varphi ! y \text{ Df}$ <sup>13</sup>

Russell does not mention Leibniz or Frege.

If we read this equality left to right as a conditional, it is Leibniz's Law – the Identity of Indiscernibles, which is a tautology, analytically true. If two things are identical, they share every property. Sharing every intrinsic and extrinsic property is only possible for a thing itself.

If we read it right to left, it is the converse of Leibniz's Law – the Indiscernibility of Identicals (this converse name suggested by Quine in 1943). This is best understood as a hypothetical and synthetic statement, its validity to be determined empirically. If we discover two things that share every property, they are identical. Leibniz was emphatic that this is not possible for numerically distinct objects. This at most is *relative identity*.

“This definition states that x and y are to be called identical when every predicative function satisfied by x is also satisfied by y. We cannot state that every function satisfied by x is to be satisfied by

12 Peirce (1902) ‘The Simplest Mathematics,’ *Collected Papers. Vol.4*, (1933), p.251

13 Russell (1927) *Principia Mathematica*, Vol. 1, Second Edition, p.176



y, because x satisfies functions of various orders, and these cannot all be covered by one apparent variable. But in virtue of the axiom of reducibility it follows that, if  $x = y$  and x satisfies  $\psi x$ , where  $\psi$  is any function, predicative or non-predicative, then y also satisfies  $\psi y$  (cf. \*13.101., below). Hence in effect the definition is as powerful as it would be if it could be extended to cover all functions of x...

The propositions of the present number are constantly referred to. Most of them are self-evident, and the proofs offer no difficulty. The most important of the propositions of this number are the following:

\*13.101.  $\vdash : x = y. \supset . \psi x \supset \psi y$

I.e. if x and y are identical, any property of x is a property of y.

\*13.12.  $\vdash : x = y. \supset . \psi x \supset \psi y$

This includes \*13.101 together with the fact that if x and y are identical any property of y is a property of x.

\*13.15.16.17. which state that identity is reflexive, symmetrical and transitive.”<sup>14</sup>

## Wittgenstein

Wittgenstein also does not mention Leibniz in his section on identity in the *Tractatus*, but the substance of Leibniz’s Law is in his 5.5302.

5.53 Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs.

5.5301 That identity is not a relation between objects is obvious. This becomes very clear if, for example, one considers the proposition “ $(x) : fx . \text{HOOK} . x = a$ ”. What this proposition says is simply that only a satisfies the function f, and not that only such things satisfy the function f which have a certain relation to a.

One could of course say that in fact only a has this relation to a, but in order to express this we should need the sign of identity itself.

5.5302 Russell’s definition of “=” won’t do; because according to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it is nevertheless significant.)

14 *Ibid.*



5.5303 Roughly speaking: to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing.

5.532 And analogously: not “( EXISTS  $x$ ,  $y$  ) .  $f(x, y)$  .  $x=y$ ”, but “( EXISTS  $x$  ) .  $f(x, x)$ ”; and not

“( EXISTS  $x$ ,  $y$  ) .  $f(x, y)$  .  $\sim x=y$ ”, but “( EXISTS  $x$ ,  $y$  ) .  $f(x, y)$ ”.

Therefore instead of Russell’s “( EXISTS  $x$ ,  $y$  ) .  $f(x, y)$ ” : “( EXISTS  $x$ ,  $y$  ) .  $f(x, y)$  .  $\vee$  . ( EXISTS  $x$  ) .  $f(x, x)$ ”.

5.533 The identity sign is therefore not an essential constituent of logical notation.

5.534 And we see that the apparent propositions like: “ $a=a$ ”, “ $a=b$  .  $b=c$  . HOOK  $a=c$ ”, “(  $x$  ) .  $x=x$ ”. “( EXISTS  $x$  ) .  $x=a$ ”, etc. cannot be written in a correct logical notation at all.

5.535 So all problems disappear which are connected with such pseudo-propositions.

This is the place to solve all the problems which arise through Russell’s “Axiom of Infinity”.

What the axiom of infinity is meant to say would be expressed in language by the fact that there is an infinite number of names with different meanings.<sup>15</sup>

## Frank Ramsey on Identity

FRANK RAMSEY criticized the section on identity in *Principia Mathematica*, He too uses Leibniz’s Law.

“The third serious defect in *Principia Mathematica* is the treatment of identity. It should be explained that what is meant is numerical identity, identity in the sense of counting as one, not as two. Of this the following definition is given:

‘  $x = y$  . = : (  $\varphi$  ) :  $\varphi ! x$  .  $\supset$  .  $\varphi ! y$  : Df. ‘ [Cf., *Principia Mathematica*, 13.01]

That is, two things are identical if they have all their elementary properties in common...

The real objection to this definition of identity is the same as that urged above against defining classes as definable classes: that it is a misinterpretation in that it does not define the meaning with which the symbol for identity is actually used.

15 Wittgenstein (1922) *Tractatus Logico-Philosophicus*, section 5.53



This can be easily seen in the following way: the definition makes it self-contradictory for two things to have all their elementary properties in common. Yet this is really perfectly possible, even if, in fact, it never happens. Take two things, a and b. Then there is nothing self-contradictory in a having any self-consistent set of elementary properties, nor in b having this set, nor therefore, obviously, in both a and b having them, nor therefore in a and b having all their elementary properties in common. Hence, since this is logically possible, it is essential to have a symbolism which allows us to consider this possibility and does not exclude it by definition.

It is futile to raise the objection that it is not possible to distinguish two things which have all their properties in common, since to give them different names would imply that they had the different properties of having those names. For although this is perfectly true—that is to say, I cannot, for the reason given, know of any two particular indistinguishable things—yet I can perfectly well consider the possibility, or even know that there are two indistinguishable things without knowing which they are.”<sup>16</sup>

For distinct objects to be identical in Ramsey’s sense, we would have to ignore relational properties and positional properties, and focus only on intrinsic properties.

Is an object’s name a property? It is certainly not intrinsic, essential or even a peculiar quality, in Aristotle’s and the Stoics’ sense.

Leibniz’s Law about the *identity of indiscernibles* is not enough. Some properties that differ might not be discernible, as he knew.

### Willard Van Orman Quine on Identity

WILLARD VAN ORMAN QUINE commented on identity in his 1940 book *Mathematical Logic*, explaining it in terms of class membership.

“WE TURN now to the problem of so defining ‘ $x = y$ ’, in terms of ‘ $\in$ ’ and our other primitives, that it will carry the intended sense ‘ $x$  and  $y$  are the same object’. In the trivial case where  $y$  is not a class, indeed,  $x \in y$  if and only if  $x = y$  in this sense (cf. § 22); but our problem remains, since ‘ $x \in y$ ’ diverges in meaning from ‘ $x = y$ ’ in case  $y$  is a class. We must find a formula, composed of ‘ $x$ ’

16 Ramsey (1926) *The Foundation of Mathematics*, p.29 in the 1960 edition





and ‘y’ by means of ‘ $\in$ ’ and our other primitives, which will be true just in case x and y are the same object — whether a class or a non-class. The requirement is met by:

$$(1) (z)(z \in x . = . z \in y)$$

when x and y are classes, since classes are the same when their members are the same (cf. § 22). Moreover, (1) continues to meet the requirement when x and y are not classes. For, in this case ‘ $z \in x$ ’ and ‘ $z \in y$ ’ identify z with x and with y; and (1) as a whole then says that whatever is the same as x is the same as y, thus identifying x and y. Both where x and y are classes and where they are not, therefore, (1) meets our requirements; (1) is true if and only if x and y are the same. We are thus led to introduce ‘ $x = y$ ’ as an abbreviation of (1)...

Variables and abstracts will be spoken of collectively as terms. Now let us supplement our Greek-letter conventions to this extent: just as we use ‘ $\varphi$ ’, ‘ $\psi$ ’, and ‘ $\chi$ ’, to refer to any formulae, and ‘ $\alpha$ ’, ‘ $\beta$ ’, ‘ $\gamma$ ’, and ‘ $\delta$ ’ to refer to any variables, so let us use ‘ $\zeta$ ’, ‘ $\eta$ ’, and ‘ $\theta$ ’ (along with their accented and subscripted variants) to refer in general to any terms. With help of this convention we can express the general definition of identity as follows, for application to variables and abstracts indifferently:

$$D10. \ulcorner (\zeta = \eta) \urcorner \text{ for } \ulcorner (\alpha) (\alpha \in \zeta . = . \alpha \in \eta) \urcorner^{17}$$

In 1943, a few years before Ruth C. Barcan introduced her two new modal operators,  $\diamond$  for possibility, and  $\square$  for necessity (the square was suggested by her thesis adviser, Frederic B. Fitch), Quine published an important paper on existence and necessity.

Here is the converse of Leibniz’s Law, first given its converse name here by Quine:

“One of the fundamental principles governing identity is that of substitutivity - or, as it might well be called, that of indiscernibility of identicals. It provides that, given a true statement of identity, one of its two terms may be substituted for the other in any true statement and the result will be true. It is easy to find cases contrary to this principle. For example, the statements:

(1) Giorgione = Barbarelli,

2) Giorgione was so-called because of his size

are true; however, replacement of the name ‘Giorgione’ by the

17 Quine (1951) §25 ‘Identity’, *Mathematical Logic*, p.134 in the 1951 edition.



name 'Barbarelli' turns (2) into the falsehood:  
Barbarelli was so-called because of his size."<sup>18</sup>

Frege had warned about the confusion possible between the bare denotation or name and the sense intended by the speaker and interpreted by the listener. C. I. LEWIS said we need to consult the intension, the meaning, to draw the right logical conclusions. Lewis felt Quine's extensionality, based on set membership, is not enough.

The proper resolution of this word quibble and quasi-paradox is to take the intension of "Barbarelli" as a second name for the same thing named by "Giorgione" - "big George." Barbarelli, *qua* Giorgione, was so-called because of his size.

In his brief discussion of necessity, Quine, following RUDOLF CARNAP, said

"Among the various possible senses of the vague adverb 'necessarily', we can single out one - the sense of *analytic* necessity - according to the following criterion: the result of applying 'necessarily' to a statement is true if, and only if, the original statement is analytic.

(16) Necessarily no spinster is married,  
for example, is equivalent to:

(17) 'No spinster is married' is analytic,  
and is therefore true."

Quine concludes that the notion of necessity may simply not be susceptible to quantification, and insists extensionality is the best approach, because there is no need for intensionality in mathematics!

The effect of these considerations is rather to raise questions than to answer them. The one important result is the recognition that any intensional mode of statement composition, whether based on some notion of "necessity" or, for example, on a notion of "probability" (as in Reichenbach's system), must be carefully examined in relation to its susceptibility to quantification. Perhaps the only useful modes of statement composition susceptible to quantification are the extensional ones, reducible to '·' and '∴'. Up to now there is no clear example to the contrary. It is known, in particular, that no intensional mode of statement composition is needed in mathematics.<sup>19</sup>

18 Quine (1943) 'Notes on Existence and Necessity,' p.113

19 *Ibid.* p.124-5



Immediately after Barcan's 1946 paper, Quine said there would be problems interpreting *quantified modal logic*. Quine himself was the source of most of those problems.

He clearly distinguished *a priori*, *analytic*, and *necessary* truths. The first include only logical signs, the second uses words and the semantics of symbolic logic. Necessity he calls modal and interprets it in terms of analyticity.

"All true statements which (like '(x) (x = x)') contain only logical signs are naturally to be classified as logically true. But there are also other logically true statements (e. g. 'Socrates is mortal  $\supset$  Socrates is mortal'). which contain extra-logical signs...

The class of analytic statements is broader than that of logical truths, for it contains in addition such statements as 'No bachelor is married.' ...

What is rather in point, I think, is a relation of synonymy, or sameness of meaning, which holds between expressions of real language (though there be no standard hierarchy of definitions. In terms of synonymy) and logical truth we could define analyticity: a statement is analytic if by putting synonyms for synonyms (e.g. 'man not married' for 'bachelor') it can be turned into a logical truth.

The particular synonymy relation wanted is one of several which have about equal right to the name "synonymy" and are all describable as "sameness of meaning" - in varying senses of "meaning." Synonymy of the kind which renders expressions interchangeable without violence to indirect quotation, for example...

We need consider only the mode of logical necessity, symbolized by ' $\square$ '; for the other modal ideas (possibility, impossibility, and the strict conditional and biconditional) are expressible in terms of necessity in obvious fashion. Now ' $\square$ ' is not quite interchangeable with 'is analytic,' for this reason: the former attaches to a statement (as ' $\sim$ ' does) to form a statement containing the original statement, whereas 'is analytic' (like 'is true,' 'is false') attaches to the name of a statement to form a statement about the named statement. Grammatically ' $\square$ ' is an adverb; 'is analytic' is a verb...

However, ' $\square$ ' can be explained in terms of analyticity as follows:



(i) The result of prefixing ‘ $\square$ ’ to any statement is true if and only if the statement is analytic.”<sup>20</sup>

Quine spent the next several years publishing examples of failure of this substitutivity of synonyms which change meaning.

Quine uses the new necessity symbol, ‘ $\square$ ’, suggested by Ruth Barcan’s thesis adviser at Yale, F. B. Fitch, and introduced in 1946.

### Max Black

In the same year that he and PETER GEACH translated Frege’s *Sinn und Bedeutung* (1952), Black wrote an amusing dialogue questioning an identity that allows  $a = b$  and his opponent suggested two spheres in otherwise empty space could be identical. He wrote:

“B. Then this is a poor way of stating your conclusion. If  $a$  and  $b$  are identical, there is just one thing having the two names ‘ $a$ ’ and ‘ $b$ ’; and in that case it is absurd to say that  $a$  and  $b$  are two. Conversely, once you have supposed there are two things having all their properties in common, you can’t without contradicting yourself say that they are “identical”.

A. I can’t believe you were really misled. I simply meant to say it is logically impossible for two things to have all their properties in common.

I showed that  $a$  must have at least two properties—the property of being identical with  $a$ , and the property of being different from  $b$ —neither of which can be a property of  $b$ . Doesn’t this prove the principle of Identity of Indiscernibles?

B. Perhaps you have proved something. If so, the nature of your proof should show us exactly what you have proved. If you want to call “being identical with  $a$ ” a “property” I suppose I can’t prevent you. But you must then accept the consequences of this way of talking. All you mean when you say “ $a$  has the property of being identical with  $a$ ” is that  $a$  is  $a$ . And all you mean when you say “ $b$  does not have the property of being identical with  $a$ ” is that  $b$  is not  $a$ . So what you have “proved” is that  $a$  is  $a$  and  $b$  is not  $a$ , that is to say,  $b$  and  $a$  are different. Similarly, when you said that  $a$ , but not  $b$ , had the property of being different from  $b$ , you were simply saying that  $a$  and  $b$  were different...

Isn’t it logically possible that the universe should have contained nothing but two exactly similar spheres? We might suppose that

<sup>20</sup> Quine (1947) ‘The Problem of Interpreting Modal Logic,’ *Journal of Symbolic Logic* (1947) 12 (2) pp.43, 45



each was made of chemically pure iron, had a diameter of one mile, that they had the same temperature, colour, and so on, and that nothing else existed.”<sup>21</sup>

Black says that *b* cannot have the self-identical property of “= *a*.” Yet we will find this in many modern arguments (e.g., Wiggins, Kripke) Black’s spheres could of course have identical *intrinsic* information. We just need to ignore their coordinates and relations to each other and say they are relatively identical.

### Ruth Barcan Marcus

In 1947, Ruth C. Barcan (later *Ruth Barcan Marcus*) wrote an article on “The Identity of Individuals.” It was the first assertion of the so-called “necessity of identity.” Her work was written in the dense expressions of symbolic logic, with little explanation. We present it here for historical completeness,

$$\begin{aligned}
 2.33^* \vdash (\beta_1 I(\beta_2) \equiv (\beta_1 I_m(\beta_2). \\
 ((\beta_1 I_m(\beta_2) (\beta_1 I(\beta_1)) \text{ hook } (\beta_1 I(\beta_2) \quad 2.21, 2.3, \text{ subst, } 14.26 \\
 (\beta_1 I_m(\beta_2) \text{ hook } (\beta_1 I(\beta_2) \quad 2.6, 2.32^*, \text{ subst, adj, } 18.61, \text{ mod pon} \\
 (\beta_1 I(\beta_2) \equiv (\beta_1 I_m(\beta_2) \quad 18.42, 2.23, \text{ subst, adj, def}
 \end{aligned}$$

Five years later, Marcus’s thesis adviser, Frederic B. Fitch, published his book, *Symbolic Logic*, which contained the simplest proof ever of the necessity of identity, by the simple mathematical *substitution* of *b* for *a* in the necessity of self-identity statement (2).

#### 23.4

$$(1) a = b,$$

$$(2) \Box [a = a],$$

then (3)  $\Box [a = b]$ , by identity elimination.<sup>22</sup>

Clearly this is *mathematically* and *logically* sound. Fitch substitutes *b* from (1), for *a* in the modal context of (2). This would be fine if these are just mathematical equations. But as Barcan Marcus knew very well from C. I. Lewis’s work on strict implication, substitutivity in statements also requires that the substitution is intensionally meaningful. In the sense that *b* is actually just *a*, substituting *b* is equivalent to keeping *a* there, as a tautology, something with no new information. To be informative and prove the necessary truth of the

21 Black (1952) ‘The Identity of Indiscernibles,’ *Mind*, 61(242), p.154

22 Fitch (1952) *Symbolic Logic*, p.164



new statement, we must know more about b, for example, that the *intrinsic* information in b is identical to that of a. And of course this is at best *relative identity* for numerically distinct objects.

Fourteen years after her original identity article, Marcus presented her work at a 1961 colloquium at Boston University attended by Quine and Kripke.

Marcus reprised the proof of her claim about the necessity of identity. She explicitly added Leibniz's Law relating identicals to indiscernibles to her argument.

“(x)(y) (x = y)  $\supset$   $\square$  (x = y)

In a formalized language, those symbols which name things will be those for which it is meaningful to assert that *I* holds between them, where ‘*I*’ names the identity relation... If ‘x’ and ‘y’ are individual names then

(1) x *I* y

Where identity is defined rather than taken as primitive, it is customary to define it in terms of indiscernibility, one form of which is

(2) x *Ind* y =<sub>df</sub> ( $\phi$ )( $\phi$ x eq  $\phi$ y)

(3) x eq y = x *I* y.”<sup>23</sup>

Statement (2) is Leibniz's Law, the indiscernibility of x from y, by definition means that for every property  $\phi$ , both x and y have that same property,  $\phi$ x eq  $\phi$ y.

## David Wiggins

DAVID WIGGINS and Peter Geach debated back and forth about the idea of “*relative identity*” for many years after Geach first suggested it in 1962. Wiggins also speculated about the so-called *necessity of identity*, which was first argued by Marcus back in 1947.

As we saw, Ruth Barcan Marcus published her original proof of the *necessity of identity* in 1947 and repeated her argument at a 1961 Boston University colloquium. Whether Wiggins knew of Marcus 1961 is not clear. He should have known of her 1947, through Quine's criticisms, perhaps. Wiggins work is similar to her 1961 derivation (which uses Leibniz's Law). Wiggins gives no credit to Marcus, a pattern in the literature for the next few decades still seen today.

23 Marcus (1961) ‘Modalities and Intensional Languages,’ *Synthese*, 13(4), p.305



Saul Kripke clearly modeled much of his derivation after Wiggins, especially his criticism of the derivation as “paradoxical”. Kripke gives no credit to Marcus and only indirectly to Wiggins for the specific steps in his argument. And we know Kripke heard Marcus present at the 1961 colloquium.

In the two columns on the right, we compare Kripke’s somewhat abbreviated derivation of the necessity of identity with Wiggins’ longer and somewhat skeptical account. Wiggins suspected that what can be shown is not “ $x = y$ ,” but merely the tautology “ $y = y$ .”

The derivation of (2) itself, via  $x$ ’s predicate ‘ $( = x)$ ’, might be blocked by insisting that when expressions for properties are formed by subtraction of a constant or free variable, then every occurrence of that constant or free variable must be subtracted. ‘ $( a = a)$ ’ would then yield ‘ $( = )$ ’, and (2) could not be derived by using ‘ $( = x)$ ’. One would only get the impotent

$$(2') (x = y) \supset (x = x. \supset . y = y).^{24}$$

Wiggins predicates the property “ $= x$ ” of  $y$ . Kripke writes this as “ $x = y$ ,” logically equivalent, but intensionally predicating “ $= y$ ” of  $x$ !

Wiggins’ note (3) is almost Kripke’s (3), but with intensional “ $y = x$ .” Wiggins needs one more step. His (4) is Kripke’s (3).

### Saul Kripke on Identity

Kripke only indirectly cites Wiggins as the source of his argument. Just after his exposition, Kripke quotes Wiggins as saying in his 1965 “Identity-Statements”

“Now there undoubtedly exist contingent identity-statements. Let  $a = b$  be one of them. From its simple truth and (5) [= (4) above] we can derive ‘ $\square ( a = b)$ ’. But how then can there be any contingent identity statements?”<sup>25</sup>

The short answer is there cannot, if we are discussing numerically distinct material objects. Kripke goes on to describe the argument about  $b$  sharing the property “ $= a$ ” of being identical to  $a$ , which information philosophy reads as merely self-identity.

24 Wiggins (1965) ‘Identity Statements,’ *Analytical Philosophy*, pp.40-41

25 Kripke (1971) ‘Identity and Necessity,’ in Munitz, Milton, *Identity and Individuation*.. p. 136



**Wiggins (1965)**

The connexion of what I am going to say with modal calculi can be indicated in the following way. It would seem to be a necessary truth that if  $a = b$  then whatever is truly ascribable to  $a$  is truly ascribable to  $b$  and vice versa (Leibniz's Law). This amounts to the principle

$$(1) (x)(y)((x = y) \supset (\phi)(\phi x \supset \phi y))$$

Suppose that identity-statements are ascriptions or predications.! Then the predicate variable in (1) will apparently range over properties like that expressed by ' $= a$ ' and we shall get as consequence of (1)

$$(2) (x) (y) ((x = y) \supset (x = x \cdot \supset \cdot y = x))$$

There is nothing puzzling about this. But if (as many modal logicians believe), there exist de re modalities of the form

$$\Box (\phi a) \text{ (i.e., necessarily } (\phi a)\text{),}$$

then something less innocent follows. If ' $= a$ ' expresses property, then ' $\Box (a=a)$ ', if this too is about the object  $a$ , also ascribes something to  $a$ , namely the property  $\Box (= a)$ . For on a naive and pre-theoretical view of properties, you will reach an expression for a property whenever you subtract a noun-expression with material occurrence (something like ' $a$ ' in this case) from a simple declarative sentence. The property

$\Box (= a)$  then falls within the range of the predicate variable in Leibniz's Law (understood in this intuitive way) and we get

$$(3) (x) (y) (x = y \supset (\Box (x = x) \cdot \supset \cdot \Box (y = x)))$$

Hence, reversing the antecedents,

$$(4) (x) (y) (\Box (x = x) \cdot \supset \cdot (x = y) \supset \Box (x = y))$$

But  $(x) (\Box (x=x))$  'is a necessary truth, so we can drop this antecedent and reach

$$(5) (x)(y)((x = y) \cdot \supset \cdot \Box (x = y))$$

**Kripke (1971)**

First, the law of the substitutivity of identity says that, for any objects  $x$  and  $y$ , if  $x$  is identical to  $y$ , then if  $x$  has a certain property  $F$ , so does  $y$ :

$$(1) (x)(y) [(x = y) \supset (Fx \supset Fy)]$$

{Note Kripke left out Wiggins' universal quantifier ( $F$ ) - for all properties.}

On the other hand, every object surely is necessarily self-identical:

$$(2) (x) \Box (x = x)$$

But

$$(3) (x)(y) (x = y) \supset [\Box (x = x) \supset \Box (x = y)]$$

is a substitution instance of (1), the substitutivity law. From (2) and (3), we can conclude that, for every  $x$  and  $y$ , if  $x$  equals  $y$ , then, it is necessary that  $x$  equals  $y$ :

$$(4) (x)(y) ((x = y) \supset \Box (x=y))$$

This is because the clause  $\Box (x = x)$  of the conditional drops out because it is known to be true.

Compare the simplicity and clarity of Marcus' thesis adviser...

**Fitch (1952)**

$$(1) a = b,$$

$$(2) \Box [a = a],$$

$$\text{then } (3) \Box [a = b],$$

by identity elimination.





“If  $x$  and  $y$  are the same things and we can talk about modal properties of an object at all, that is, in the usual parlance, we can speak of modality *de re* and an object necessarily having certain properties as such, then formula (1), I think, has to hold. Where  $x$  is any property at all, including a property involving modal operators, and if  $x$  and  $y$  are the same object and  $x$  had a certain property  $F$ , then  $y$  has to have the same property  $F$ . And this is so even if the property  $F$  is itself of the form of necessarily having some other property  $G$ , in particular that of necessarily being identical to a certain object. [viz.,  $= x$ ]

Well, I will not discuss the formula (4) itself because by itself it does not assert, of any particular true statement of identity, that it is necessary. It does not say anything about statements at all. It says for every object  $x$  and object  $y$ , if  $x$  and  $y$  are the same object, then it is necessary that  $x$  and  $y$  are the same object. And this, I think, if we think about it (anyway, if someone does not think so, I will not argue for it here), really amounts to something very little different from the statement (2). Since  $x$ , by definition of identity, is the only object identical with  $x$ , “ $(y)(y = x \supset Fy)$ ” seems to me to be little more than a garrulous way of saying ‘ $Fx$ ’ and thus  $(x)(y)(y = x \supset Fx)$  says the same as  $(x)Fx$  no matter what ‘ $F$ ’ is — in particular, even if ‘ $F$ ’ stands for the property of necessary identity with  $x$ . So if  $x$  has this property (of necessary identity with  $x$ ), trivially everything identical with  $x$  has it, as (4) asserts. But, from statement (4) one may apparently be able to deduce various particular statements of identity must be necessary and this is then supposed to be a very paradoxical consequence.”<sup>26</sup>

The indiscernibility of identicals claims that if  $x = y$ , then  $x$  and  $y$  must share all their properties, otherwise there would be a discernible difference. Now Kripke argues that one of the properties of  $x$  is that  $x = x$ , so if  $y$  shares the property of ‘ $= x$ ,’ we can say that  $y = x$ . Then, necessarily,  $x = y$ . But this is nonsense for distinct objects.

Two distinct things,  $x$  and  $y$ , cannot be identical, because there is some difference in *extrinsic* external information between them. Instead of claiming that  $y$  has  $x$ ’s property of being identical to  $x$  (“ $= x$ ”), we can say only that  $y$  has  $x$ ’s property of being *self-identical*, thus  $y = y$ . Wiggins calls this result “impotent.” Then  $x$  and  $y$  remain distinct in at least this *intrinsic* property as well as in *extrinsic* properties like their distinct positions in space.



## Peter Geach on Relative Identity

PETER GEACH proposed the relativity of identity in 1962 and debated for years with David Wiggins about it.

For Geach and Wiggins, relative identity means “x is the same F as y,” but “x may not be the same G as y.” Wiggins argued against this idea of relative identity, but accepted what he called a sortal-dependent identity, “x is the same F as y.” Geach called this a “criterion of identity.”

“I had here best interject a note on how I mean this term ‘criterion of identity’. I maintain that it makes no sense to judge whether x and y are ‘the same’, or whether x remains ‘the same’, unless we add or understand some general term—”the same F”. That in accordance with which we thus judge as to the identity, I call a criterion of identity; this agrees with the etymology of “criterion”. Frege sees clearly that “one” cannot significantly stand as a predicate of objects unless it is (at least understood as) attached to a general term; I am surprised he did not see that the like holds for the closely allied expression ‘the same.’”<sup>27</sup>

In his 1967 article “Identity,” in the *Review of Metaphysics*, Geach wrote

“I am arguing for the thesis that identity is relative. When one says “x is identical with y”, this, I hold, is an incomplete expression; it is short for “x is the same A as y”, where “A” represents some count noun understood from the context of utterance—or else, it is just a vague expression of a half-formed thought. Frege emphasized that “x is one” is an incomplete way of saying “x is one A, a single A”, or else has no clear sense; since the connection of the concepts one and identity comes out just as much in the German “ein und dasselbe” as in the English “one and the same”, it has always surprised me that Frege did not similarly maintain the parallel doctrine of relativized identity, which I have just briefly stated. On the contrary, Frege actually enunciated with all vigour a doctrine that identity cannot be relativized: “Identity is a relation given to us in such a specific form that it is inconceivable that various forms of it should occur” (*Grundgesetze*, Vol. II, p. 254).”<sup>28</sup>

27 Geach (1962) *Reference and Generality*, p.39; 1980, p.63

28 Geach (1967) ‘Identity’, *Review of Metaphysics*, in *Logic Matters*, 1972, p.238-



## David Lewis

DAVID LEWIS, the modern metaphysician who built on Leibniz' possible worlds to give us his theory of "modal realism," is just as clear as Leibniz on the problem of identity.

"[W]e should not suppose that we have here any problem about identity. We never have. Identity is utterly simple and unproblematic. Everything is identical to itself; nothing is ever identical to anything else except itself. There is never any problem about what makes something identical to itself, nothing can ever fail to be. And there is never any problem about what makes two things identical; two things never can be identical."<sup>29</sup>

Except, says an information philosopher, "in some respects," in which case we have *relative identity*.

## Relative Identity

The concept of relative identity, identical in some respect, identical *qua*, is a property of so-called "interchangeable parts." They can be substituted for one another. The concept of substitutability is an essential concept in mathematics, in symbolic logic, and to some extent in language, where it has generated much confusion. The fundamental ambiguity and polysemy of language, which generates its metaphorical power, means that one word or phrase is never perfectly substitutable for another.

After accepting the fundamental fact that nothing is perfectly identical to anything but itself, the criterion for relative identity, for identical "in some respect," or *qua* that respect, is that some subset of the information in two different things must be the same information, bit for bit.

We defined *I* as the sum of all the intrinsic properties and relations - internal self-relations between an object's different parts. And we defined *E* for an object as the sum of extrinsic relations an object has with things outside, including its disposition in space and time.

Relative identity means that *a* can be the same *I* as *b*, but not the same *E* as *b*. For physical objects, these could be within some physical boundary, subject to conditions of vagueness. In a biological

29 Lewis (1986) 'Counterparts or Double Lives,' *On the Plurality of Worlds*, p.192.



cal entity, it includes the vast teleonomic communications going on inside and between the cells, which makes it much more than a mereological sum of its parts.

Set theoretically, in classical propositional calculus, we can say that  $Ia$  is the set of *intrinsic* properties and relations that can be predicated in propositions about an object  $a$ .  $Ea$  is the set of *extrinsic* relations. We can now describe why absolute identity is limited to self-identity.

If  $Ia + Ea = Ib + Eb$ , then  $a$  and  $b$  are one and the same object.

And, if  $Ia = Ib$ , then  $a$  and  $b$  are relatively identical, *qua* their information content.

Metaphysicians like the notion of kinds or sorts, or even tropes, which are *abstract entities* that can be used as particular properties. All three of these can be redescribed in information terms. To be of such-and-such a sort, for example, would be to contain the information characteristic of that sort. Numerically distinct entities can then be identical in respect of being of the same sort – identical *qua* that sort.

Seeing the relative identity between two things is something done by minds. This is a mind's ability to "pick out" the resemblances. The metaphysicist emphasizes resemblances that are mind-independent properties in the objects themselves. But concepts especially are always initially invented by humans and must be scrutinized for the genetic fallacy.

When information philosophy claims we have knowledge of something (in a mind), it is the claim that what is in the mind is relatively identical to some of the information in the thing. This idea has been criticized as the "picture theory of meaning." Consider Wittgenstein,

"A picture is a model of reality.

There must be something identical in a picture and what it depicts, to enable the one to be a picture of the other at all."<sup>30</sup>

The *experience recorder and reproducer* (ERR) explains the indirect way in which this happens. The perception of an object is

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30 Wittgenstein (1922) *Tractatus Logico-Philosophicus*, 2.12, 2.161



encoded in the brain as an experience. When the reproducer “plays back” the experience, the neurons that were “wired together” during earlier experiences now “fire together” and the brain presents (represents) to the mind parts of the original perception. The “decoding” process may activate any or all of the original sensations of the experience, together with any emotions recorded.

This does not mean that the information stored in the neurons is directly isomorphic to some of the information in the thing itself. Very little in the brain “resembles” the world. Exceptions are mappings of our sensorimotor apparatus, and in some animals, maps of their environment. What the ERR means is that the mind re-experiences some subset of the original experiences. This is actually very close to Wittgenstein’s “picture.” The “mind’s eye” sees before it a “representation.” ARTHUR SCHOPENHAUER called it a *Vorstellung*.

There is of course an implicit complicated mapping between neurons and the organs of sensation, somewhat analogous to the complex mapping of bits in a DVD to the colored pixels of a video monitor. But the ERR model goes well beyond a visual picture, since the body experiences a subset of the feelings that were recorded along with the original experience.

Minds not only pick out relative identity, they also see differences, so we have this apparent contradiction as first enunciated by CHARLES SANDERS PEIRCE:

“Everything is both similar and dissimilar to everything else,”<sup>31</sup>

We unpack Peirce in our three axioms as follows...

*Id1. Everything is identical to everything else in some respects.*

*Id2. Everything is different from everything else in some other respects.*

*Id3. Everything is identical to itself in all respects at each instant of time, but different in some respects from itself at any other time.*

We can rewrite these axioms in terms of information philosophy

*I1. Any two things have some information in common.*

*I2. Any two things have some different information.*

*I3. The identity of anything over time is changing because the information in it (and about it) is changing with time.*

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31 Peirce. *Collected Papers* Vol. I, Principles of Philosophy, 1.566



These three observations might be called information axioms. Armed with them, we are in a position to “dis-solve” or deconstruct some of the most famous metaphysical puzzles and paradoxes.

Now I3 requires the metaphysical possibility that information can change with time. The cosmological observation of astronomical objects provides convincing evidence of increases in the total information with time, as does biological evolution.

DAVID HUME argued that there are only three basic relations between things, contiguity, causality, and resemblance. We can see the first as how things or events are arranged in space, the second as to how they follow one another in time, the third as similarities in their form. Information philosophy condenses these three to information in space and time.

$A = A$

The mathematical expression “A equals A” (notice there are two distinct A’s) is an empty tautology. Its usefulness comes from other equivalences, such as the equation “A = B.” Whenever A appears, we may substitute B.

A and B are substitutable, interchangeable parts, for some practical purpose, like logic, mathematics, or engineering.

But, when we think and speak carefully, neither in metaphysics nor in ordinary language do we unconditionally accept the statement “A is identical to B.”

Indeed, we see that the expressions “A = A” or “A is A” are not at all innocently true, since there are manifold differences between the two A’s, their positions in space, their ink particles on the paper they are printed on, the pixels on your computer screen, etc.

It is the immaterial information content of “A,” abstracted from concrete examples of letters, that has a self-identical property, but only in the realm of information and abstract entities. Any single concrete example of an “A” has the property of self-identity, but only in the realm of material, and then only for an instant of time, because everything in the material realm is constantly changing.

Analytical language philosophers, puzzling over statements like “A is B,” say that the identity of the two symbols is because they refer



to the same thing. Much philosophical ink has been spilled puzzling over Frege's observation that "the morning star is the evening star."

The total meaningful content of this sentence is not limited to the banal point that two names or designators ("Phosphorus" and "Hesperus") are references (Frege's *Bedeutung*) to the one planet Venus (a concrete entity). We might call this property "referential identity."

While the statement "the morning star is the morning star" is considered analytically true (like "A is A"), the two terms in the statement have different meanings or senses (Frege's *Sinn*).

Information philosophers agree that the meanings of the referring terms contain much more knowledge than just the information in the planet itself. Both terms tell us where Venus is in the sky, where it is compared to the Sun along the ecliptic, when to look for it, etc. But this additional (and differing) information makes paradoxical even analytic linguistic identity.

Indeed the paradox of all analytic philosophy (that all analysis is either trivial or false) can be seen in the fact that all analytic statements are tautologies. If the expression to be analyzed (the *analysandum*) and the analyzing expression (the *analysans*) contain identical information, then the analysis is trivial.

If the *analysandum* and *analysans* do not contain the same information, the analysis is false. WILLARD VAN ORMAN QUINE threw up his hands and declared (correctly) that all knowledge must be synthetic *a posteriori* (based on experience).

### Identity through Time

Because all material things change in time (the Heraclitean "flux"), "identity over time" is fundamentally impossible. Even in the case of a hypothetical completely inert object that could be protected from loss or gain of a single atom, its external dispositional relations (e.g., position coordinates in most spacetime frames) are constantly changing, and these are fundamental "properties", in both classical Aristotelian and modern Kantian categories.

If we identify the essence of something as the total information that makes it identical with itself, then all that information is essential. Several puzzling metaphysical facts follow that do violence to our ordinary way of talking about essence and identity.



Aristotle's distinction between essence or Being ( $\tau\acute{o} \delta\upsilon\nu$ ) and accident ( $\sigma\upsilon\mu\beta\epsilon\beta\epsilon\kappa\acute{o}\varsigma$ ) surely did not make every property or quality of an entity essential. But modern metaphysicians do argue for a number of "essentialisms." We shall see that they are mostly the result of the metaphysicians' definitions. They are in no way "true at any world" in the sense of a "mind-independent" external world, let alone facts in our world, except for their arbitrary definitions.

### Changes in Time

However imperceptibly, every concrete material thing changes both its matter and form with time. The Heraclitean river changes its water constantly at any particular place. Living things change their material elements very rapidly as they ingest low-entropy, high information food and excrete higher-entropy, lower information matter.

It is only immaterial abstract entities that do not change. They have Parmenidean "Being."

Something that changes in time cannot be perfectly identical to what it was in the past. If it were identical, there would be no change. This gives rise to several metaphysical problems that involve different persistence conditions for different properties of an entity.

Information philosophy shows the way out of this apparent paradox by distinguishing the part or parts of information that are changing from any part which is constant. We can then say that an entity is identical to its earlier self "with respect to" (or "qua") the unchanged information.

What emerges is the concept of a relative or partial identity over time, accompanied by partial or relative differences in the object.

We have seen that change can be in the intrinsic or internal properties of a thing, or in its extrinsic relations to external objects, its dispositional properties such as its coordinates. The primary view of change is a real, metaphysical change in a "thing itself." Some metaphysicians argue that this must be a change of identity.

The conservation of matter and energy requires that there cannot be complete destruction of an entity and creation of a new entity from nothing. But identity never changes completely, because





modest changes in the material substrate or the information content (shape and form, internal and external relations and communications) do not invalidate an essential relative identity over time of any object.

Because of motion and microscopic physical events, all material things change in time. Change in time means that the concept of “perfect or strict identity over time” is fundamentally flawed. Even in the case of a hypothetical completely inert object that could be protected from loss or gain of a single particle, its position coordinates in most spacetime frames are constantly changing. All the other objects in the universe are changing their spatial relations with the object.

### Personal Identity

Apart from the obvious fact that every person (individual) is different from every other person, which has been confirmed by the latest understanding of all biological organisms, even an individual person is not perfectly identical to her or his self over time.

If persons were perfectly identical to themselves over time, they would not experience growth, one of the defining, therefore essential, characteristics of living things. Moreover, some metaphysicians who claim that material constitution is identity maintain that even the loss or gain of a tiny bit of matter destroys an individual and replaces that individual with another. This is a flawed idea put forward by the ancient Skeptics still taught in modern metaphysics.<sup>32</sup>

### Identity and Biology

Since the creation of information and its communication is the outstanding characteristic of life, biological information is perhaps the best way to explain the relative identity, the persistence of living things through time, *qua* person, for example. An information-based metaphysics can help solve the problem of personal identity. The genetic code (DNA) remains essentially constant through the life of an individual and should be mentioned first as a uniquely “identifying” piece of information.

Besides this “Evo” element, there is information that is created and preserved during an individual’s growth and development (the

<sup>32</sup> See chapter 27 on the growing argument.



“Devo” element). For higher organisms especially, this is its ability to record its past experiences and play them back as a guide to present actions. The experience recorder and reproducer (ERR) is a central component of consciousness and memory. This is the psychological argument for the persistence of personal identity.

### Vague Identity

The primary source of vagueness in philosophy has been vagueness in the language terms used to identify an object, which lack the information content or depth to match the information depth in typical physical objects, let alone living things.

Ontological vagueness in the position of things themselves is introduced by the uncertainty principle of quantum mechanics.

There is a deep metaphysical connection between vagueness and possibilities. An object or event that has more than one possible future can be said to be vague not in the usual spatial sense or mereological sense, but in the temporal sense.

The bit-by-bit nature of digital information introduces vagueness in the representation of analog (continuous) objects, if there are any. Whether the nature of fundamental reality shows matter to be analog or digital, fields or particles, is a deep metaphysical question.

