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Frege's Puzzle

In his 1879 *Begriffsschrift* (or “Concept-Writing”), GOTTLOB FREGE developed a propositional calculus to determine the truth values of propositions from their general form, not from any particular predicates (using specific words, names, properties, attributes, relations, etc.) The propositional calculus, a truth-functional analysis of statements as a whole, is widely considered to be the greatest advance in logic since ARISTOTLE, whose logic of syllogisms was a predicate logic, where truths depend on the meaning of individual terms in the predicate (or the subject).

In Frege's 1892 *Über Sinn und Bedeutung* (“Sense and Reference”), he distinguished the reference (name, denotation, extension, signifier) from the sense (meaning, connotation, intension, significance). He called the reference “direct” and the sense “indirect.” Frege was very clear about how the *Bedeutung*, literally the pointing out or indication of an object or concept, generates different ideas in the minds of different persons.

He says that all persons probably get a basic “sense” of a reference, from the common knowledge of things passed down through the generations, but that the particular ideas, or representations (*Vorstellung*) in each mind will be different, because everyone has had a different set of experiences, different memories. This agrees perfectly with our idea of an *experience recorder and reproducer* (ERR). Particular “meanings” are dependent on what a given mind plays back when stimulated by a new experience. Frege said ideas could only be compared if they were both present to the same consciousness, which is of course impossible.

What is sometimes called Frege's Puzzle is how two names for the same object can be distinct words (his example was the Morning Star and Evening Star) and yet in some respect be identical? His word was *Gleichheit* (“sameness”), mistranslated into English as identity by Peter Geach.

Here begins a vast problematic in philosophy that persists for the next one hundred and thirty-five years. Frege speculated that



two references to the same object could therefore be considered “identical” *in that respect* even if the “names” are distinct.

Frege was following GOTTFRIED LEIBNIZ, who said, “To suppose two things indiscernible is to suppose the same thing under two names.” Here is how Frege described it...

“Sameness gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In my *Begriffsschrift* I assumed the latter. The reasons which seem to favor this are the following: $a = a$ and $a = b$ are obviously statements of differing cognitive value; $a = a$ holds a priori and, according to Kant, is to be labeled analytic, while statements of the form $a = b$ often contain very valuable extensions of our knowledge and cannot always be established a priori...

Now if we were to regard sameness as a relation between that which the names “a” and “b” refer to, it would seem that $a = b$ could not differ from $a = a$ (i.e., provided $a = b$ is true).¹

A relation would thereby be expressed of a thing to itself, and indeed one in which each thing stands to itself but to no other thing. What is intended to be said by $a = b$ seems to be that the signs or names “a” and “b” refer to the same thing, so that those signs themselves would be under discussion; a relation between them would be asserted. But this relation would hold between the names or signs only insofar as they named or designated something. It would be mediated by the connection of each of the two signs with the same designated thing. But this is arbitrary. Nobody can be forbidden to use any arbitrarily producible event or object as a sign for something. In that case the sentence $a = b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means. But in many cases this is just what we want to do. If the sign “a” is distinguished from the sign “b” only as object (here, by means of its shape), not as sign (i.e., not by the manner in which it designates something), the cognitive value of $a = a$ becomes essentially equal to that of $a = b$, provided $a = b$ is true. A difference can arise only if the difference between the signs corresponds to a difference in the

¹ This works in mathematics, as Fitch and Quine saw, but not in ordinary language. See p.158.



mode of presentation of that which is designated...

If we found “ $a = a$ ” and “ $a = b$ ” to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its referent, i.e., its truth value. If now $a = b$, then indeed the referent of “ b ” is the same as that of “ a ,” and hence the truth value of “ $a = b$ ” is the same as that of “ $a = a$.”

In spite of this, the sense of “ b ” may differ from that of “ a ,” and thereby the sense expressed in “ $a = b$ ” differs from that of “ $a = a$.” In that case the two sentences do not have the same cognitive value.”²

Names and Reference

Frege's puzzle³ is clear, the names “ a ” and “ b ” refer to the same thing, but the names are only identical *qua* references to the object. They may have different senses, or meanings.

Since Frege, generations of philosophers have puzzled over different names and/or descriptions referring to the same thing that may lead to logical contradictions when one term is *substituted* for the other in a logical statement. Frege's original example was the Morning Star and Evening Star (often called Hesperus and Phosphorus) as names that refer to the planet Venus. Do these names have differing cognitive value? Yes. Can they be defined *qua* references to uniquely pick out Venus. Yes.

The names are relations, words that are references to the objects. But words put us back into the ambiguous realm of language.

Over a hundred years of confusion in logic and language consisted of finding two expressions that can be claimed in some sense to be identical, but upon substitution in another statement, they do not preserve the truth value of the statement. Besides Frege, and a few examples from BERTRAND RUSSELL (“Scott” and “the author of *Waverly*,” “bachelor” and “unmarried man”), WILLARD VAN ORMAN QUINE was the most prolific generator of paradoxes (“9” and “the number of planets,” “Giorgione” and “Barbarelli,” “Cicero” and “Tully,” and others).

2 Frege (1892) in Geach and Black (1952) *Sense and Reference*, *Translations from the Philosophical Writings of Gottlob Frege*, pp.209, 230

3 Salmon (1986) . *Frege's Puzzle*.



Just as information philosophy shows how to pick out information in an object or concept that constitutes the “peculiar qualifications” that individuate it, so we can pick out the information in two designating references that provide what Quine called “purely designative references.” Where Quine picks out information that leads to contradictions and paradoxes (he calls this “*referential opacity*”), we can “qualify” the information needed to make the terms referentially transparent.

Quine’s Paradoxes

Quine generated a number of apparently paradoxical cases where truth value is not preserved when “quantifying into a modal context.” But these can all be understood as a failure of substitutivity of putatively identical entities. Information philosophy shows that two distinct expressions that are claimed to be identical are never identical in all respects. So a substitution of one expression for the other may not be identical in the relevant respect. Such a substitution can change the meaning, the intension of the expression.

Perhaps Quine’s most famous paradox is his argument about the number of planets:

(1) 9 is necessarily greater than 7

for example, is equivalent to

‘9 > 7’ is analytic

and is therefore true (if we recognize the reducibility of mathematics to logic)...⁴

Given, say that

(2) the number of planets is 9,

we can substitute ‘the number of planets’ from the non-modal statement (2) for ‘9’ in the modal statement (1), which gives us the false modal statement

(3) The number of planets is necessarily greater than 7

But this is false, says Quine, since the statement,

(2) The number of planets is 9,

is true only because of circumstances outside of logic.

4 Quine (1943) ‘Notes on Existence and Necessity,’ p.121



RUTH BARCAN MARCUS analyzed this problem in 1961, which she calls the “familiar example” :

“(27) 9 eq the number of planets

is said to be a true identity for which substitution fails in

(28) $\Box(9 > 7)$

for it leads to the falsehood

(29) $\Box(\text{the number of planets} > 7)$.

Since the argument holds (27) to be contingent ($\sim \Box(9 \text{ eq the number of planets})$), ‘eq’ of (27) is the appropriate analogue of material equivalence and consequently the step from (28) to (29) is not valid for the reason that the substitution would have to be made in the scope of the square.”⁵

This *failure of substitutivity* can be understood by unpacking the use of “the number of planets.” It is not a *purely designative* reference, as Quine calls it.

In (27), “the number of planets” is the empirical answer to the question “how many planets are there in the solar system?” It is not what SAUL KRIPKE would call a “rigid designator” of the number 9. The intension of this expression, its reference, is the “extra-linguistic” fact about the quantity of planets (which Quine appreciated).

The expression ‘9’ is an unambiguous mathematical (logical) reference to the number 9. It refers to the number 9, which is its meaning (intension). Kripke mistakenly claims that ‘9’ is a rigid designator of the number 9 “in all possible worlds.” This is false. Only the mathematical concept of the number 9 is true in all possible worlds, not its name.

We can conclude that (27) is not a true identity, unless before “the number of planets” is *quantified*, it is *qualified* as “the number of planets *qua* its numerosity, as a pure number.” Otherwise, the reference is “opaque,” as Quine describes it. But this is a problem of his own making.

As Marcus says, when we recognize (27) as contingent, $\sim \Box(9 \text{ eq the number of planets})$, it is not necessary that 9 is equal to the number of planets, its reference to the number 9 becomes opaque. Indeed, today there are only eight planets, proving (27) was contingent.

5 Marcus (1961) “Modalities and Intensional Languages,” p. 313



The substitution of a possible or contingent empirical fact that is not “true in all possible worlds” for a logical-mathematical concept that is necessarily true is what causes the substitution failure.

When all three statements are “in the scope of the square” (\square), when all have the same modality, we can “quantify into modal contexts,” as Quine puts it. Both expressions, ‘9’ and ‘the number of planets, *qua* its numerosity,’ will be references to the same thing,

They will be identical in one respect, *qua* number. They will be “referentially transparent.”

The New Theory of Reference⁶

Frege’s Puzzle motivated several philosophers to develop a new theory of how words refer to objects, especially in modal contexts. It gave rise to Saul Kripke’s theories about “possible world semantics.”

When in the 1940’s, RUTH C. BARCAN and RUDOLF CARNAP added modal operators to quantification theory, Quine strongly objected, developing his demonstrations that “quantifying into modal contexts” leads to “referential opacity” and logical nonsense like “the number of planets is necessarily greater than 7.”

This was nothing but the fact first seen by Frege that different descriptions, different names that are “disguised descriptions,” have different cognitive value, different “senses,” that cannot be substituted for one another in any logical context, not just modal contexts, as Quine thought.

What we call a “concept” about a material object is some subset of the information in the object, accurate to the extent that the concept is isomorphic to that subset. By “picking out” different subsets, we can sort objects. We can compare objects, finding them similar *qua* one concept and different *qua* another concept. We can say that “a = b” *qua* color but not *qua* size.

Frege said that “the Morning Star = the Morning Star” is an identity and therefore tautological and tells us nothing. But “the Morning Star = the Evening Star” has additional cognitive value. In 1961, Ruth Barcan Marcus said it tells us something empirical about Venus in the morning and evening skies. She suggested less ambiguous,

6 See Humphreys and Fetzer (1999) *The New Theory of Reference*.



purely designative names would have no cognitive value beyond their reference to named objects.

Her work gave rise to the sophisticated but problematic modern idea of the “necessity of identity.”⁷

In modern times, Frege's insight has been defended with elaborate modal logical arguments, beginning with Barcan (later Marcus) in 1947, using Leibniz's Law about identity and indiscernibility, that seem to suggest that for any a and b , if $a = b$ (even contingently), then necessarily $a = b$.

$$\forall x \forall y (x = y) \supset [\Box(x = x) \supset \Box(x = y)]$$

This “indiscernibility of identicals” claims that if $x = y$, then x and y must share all properties, otherwise there would be a discernible difference. SAUL KRIPKE, following Marcus but not mentioning her, argues that one of the properties of x is that $x = x$, so if y shares the property of ‘ $= x$,’ we can say that $y = x$. Then, necessarily, $x = y$.

However, two distinct things, x and y , cannot be identical, because there is some difference in information between them. Instead of claiming that y has x 's property of being identical to x , information philosophy can say only that y has x 's property of being self-identical, thus $y = y$.

Then x and y remain distinct in at least this intrinsic property as well as in extrinsic properties like their distinct positions in space.

DAVID WIGGINS' eventually gave credit to Barcan Marcus, “Miss Barcan's proof was long received with incredulity by those committed to the mutual assimilation (much criticized in more recent times by Kripke and others) of the categories of necessity and *a priori*, and rejected on the grounds that the identity of evening and morning star was an *a posteriori* discovery. But even if statement ascertainable *a priori* to be true and necessary true statement coincided perfectly in their extensions, Miss Barcan's theorem could still stand in our version. For the conclusion is not put forward here as a necessarily true statement. (On this we remain mute.)”⁸

7 See chapter 13 on Identity

8 Wiggins (1980) *Sameness and Substance*, pp. 110-111

